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Exercise Set 13

Exercise 13.1. Let $A \in \{0,1\}^{m \times n}$ be a matrix where each row contains exactly one block of consecutive 1's, i.e. for each row i there are l_i and r_i such that $a_{ij} = 1$ if and only if $l_i \leq j \leq r_i$. Show that A is totally unimodular.

(3 points)

Exercise 13.2. Show that the incidence matrix of a graph G is totally unimodular if and only if G is bipartite. Give a proof of Kőnig's Theorem using this fact.

(2+2 points)

Exercise 13.3. Give integral polymatroids $P(f)$, $P(g)$, and $P(h)$ such that their intersection $P(f) \cap P(g) \cap P(h)$ is not integral.

(5 points)

Exercise 13.4. Consider the LP relaxation of the Survivable Network Design Problem described in the lecture. Assume that f is obtained from an instance of the SNDP rather than being an arbitrary proper function. Show that the SNDLP can be formulated as an LP of polynomial size in this case. Specifically, show that there is an LP of polynomial size with variables $(x_e)_{e \in E(G)} \cup X$ such that $x^* \in \mathbb{R}^{E(G)}$ is feasible for the SNDLP if and only if there is $y^* \in \mathbb{R}^X$ such that (x^*, y^*) is feasible for your LP.

(4 points)

Note: This is the last exercise sheet relevant for exam admissions.

Deadline: Jan 21$^{\text{st}}$, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ws24/co_exercises_ws.html`

In case of any questions feel free to contact me at mkaul@uni-bonn.de.