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Exercise Set 6

Exercise 6.1. Let G be an undirected graph and $b_1, b_2 \colon V(G) \to \mathbb{Z}_{\geq 0}$. Describe the convex hull of functions $f \colon E(G) \to \mathbb{Z}_{\geq 0}$ with $b_1(v) \leq \sum_{e \in \delta(v)} f(e) \leq b_2(v)$.

Hint: For $X, Y \subseteq V(G)$ with $X \cap Y = \emptyset$ consider the constraint

$$\sum_{e \in E(G[X])} f(e) - \sum_{e \in E(G[Y]) \cup E(Y,Z)} f(e) \leq \left\lfloor \frac{1}{2} \left(\sum_{x \in X} b_2(x) - \sum_{y \in Y} b_1(y) \right) \right\rfloor,$$

where $Z := V(G) \setminus (X \cup Y)$.

(5 points)

Exercise 6.2. Given an undirected graph G and disjoint sets $S_e, S_o \subseteq V(G)$, a *partial* (S_e, S_o)*-join* is a set $J \subseteq E(G)$ such that $|\delta(v) \cap J|$ is even for every $v \in S_e$ and odd for every $v \in S_o$. (In particular, a T-join is the same as a partial $(V(G) \setminus T, T)$-join.) Consider the Minimum Weight Partial (S_e, S_o)-Join Problem: Given an undirected graph G with edge-weights $c : E(G) \to \mathbb{R}_{\geq 0}$ and disjoint sets $S_e, S_o \subseteq V(G)$, find a partial (S_e, S_o)-join of minimum weight, or determine that none exists. Show that this problem can be linearly reduced to the Minimum Weight T-Join Problem.

(5 points)

Exercise 6.3. Let $\lambda_{ij}, 1 \leq i, j \leq n$, be nonnegative numbers with $\lambda_{ij} = \lambda_{ji}$ and $\lambda_{ik} \geq \min\{\lambda_{ij}, \lambda_{jk}\}$ for any three distinct indices $i, j, k \in \{1, \ldots, n\}$. Show that there exists a graph G with $V(G) = \{1, \ldots, n\}$ and capacities $u \colon E(G) \to \mathbb{R}_+$ such that the local edge-connectivities are precisely the λ_{ij}.

Hint: Consider a maximum weight spanning tree in (K_n, c), where $c(\{i, j\}) := \lambda_{ij}$.

(4 points)

Deadline: November 19^{th}, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ws24/co_exercises_ws.html`

In case of any questions feel free to contact me at mkaul@uni-bonn.de.