Professor Dr. Stefan Hougardy
Jannik Silvanus

Exercise Set 8

Exercise 8.1. Let G be a graph. A *2-cover* of G is a function $y : V(G) \to \{0, 1, 2\}$ with $y(v) + y(w) \geq 2$ for all $\{v, w\} \in E(G)$. The *size* of y is $\sum_{v \in V(G)} y(v)$.

If y is a 2-cover, the set $\{v \in V(G) : y(v) = 0\}$ is a stable set.
Conversely, a stable set $A \subseteq V(G)$ determines a 2-cover y by setting

$$y(v) = \begin{cases} 0 & \text{if } v \in A, \\ 2 & \text{if } v \in N(A), \\ 1 & \text{otherwise.} \end{cases}$$

Prove:

(i) The maximum size of a 2-matching in G equals the minimum size of a 2-cover of G, where the *size* of a 2-matching $f : E(G) \to \{0, 1, 2\}$ is $\sum_{e \in E(G)} f(e)$.

(ii) G has a perfect 2-matching iff $|N(A)| \geq |A|$ for all stable sets $A \subseteq V(G)$.

(4 points)

Exercise 8.2. Let G be a graph, $b : V(G) \to \mathbb{N}$, and $c : E(G) \to \mathbb{R}$ a weight function.

(i) Show that the uncapacitated maximum-weight b-matching problem in bipartite graphs can be solved in strongly polynomial time.

(ii) Use Step (i) to show that the uncapacitated maximum-weight b-matching problem can be solved in strongly polynomial time if b is even.

(iii) Use Step (ii) to show that the uncapacitated maximum-weight b-matching problem can be solved in strongly polynomial time.

(iv) Use Step (iii) to show that the capacitated maximum-weight b-matching problem for edge capacities $u : E(G) \to \mathbb{N} \cup \{\infty\}$ can be solved in strongly polynomial time.

(8 points)

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Exercise 8.3. Let G be an undirected graph and $T \subseteq V(G)$ with $|T| = 2k$ even. Prove that the minimum cardinality of a T-cut in G equals the maximum of $\min_{i=1}^{k} \lambda_{s_i,t_i}$ over all pairings $T = \{s_1, t_1, \ldots, s_k, t_k\}$, where $\lambda_{s,t}$ denotes the maximum number of pairwise edge-disjoint s-t-paths.

(4 points)

Deadline: December 7th, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ws17/co_exercises/exercises.html`

In case of any questions feel free to contact me at silvanus@or.uni-bonn.de.