Professor Dr. Stefan Hougardy
Jannik Silvanus

Exercise Set 1

Exercise 1.1. Find an infinite counterexample to Hall's Theorem. More precisely: Find a bipartite graph $G = (A \dot{\cup} B, E)$ with $A \cong \mathbb{N}$, $B \cong \mathbb{N}$, and $|N(S)| \geq |S|$ for every $S \subseteq A$ and every $S \subseteq B$ such that G does not contain a perfect matching.

(4 points)

Exercise 1.2. Let G be a bipartite graph.

(a) Let $V(G) = A \dot{\cup} B$ be a bipartition of G.
If $A' \subseteq A$ and $B' \subseteq B$, and there are a matching $M_{A'}$ covering A' and a matching $M_{B'}$ covering B', show that there must be a matching that covers $A' \cup B'$.

(b) Suppose that for every non-empty $E' \subseteq E(G)$ we have $\tau(G - E') < \tau(G)$. Show that $E(G)$ is a matching in G.

(3+1 points)

Exercise 1.3. Let G be a bipartite graph. For each $v \in V(G)$, let $<_v$ be a linear ordering of $\delta(v)$. Prove that there is a matching $M \subseteq E(G)$ such that for each $e \in E(G) \setminus M$ there is an edge $f \in M$ and a vertex $v \in V(G)$ such that $v \in (e \cap f)$ and $e <_v f$.

(4 points)

Exercise 1.4. Let G be a 3-regular undirected graph.

(a) Assume G is simple. Show that there is a matching in G covering at least $(7/8) \cdot |V(G)|$ vertices.

(b) Give an example to prove that the bound of item (a) is tight.

(c) Show that the assumption that G is simple in item (a) is necessary.

(2+1+1 points)

Professor Dr. Stefan Hougardy
Jannik Silvanus

Deadline: October 19th, before the lecture. The websites for lecture and exercises can be found at

`http://www.or.uni-bonn.de/lectures/ws17/coex.html`

In case of any questions feel free to contact me at silvanus@or.uni-bonn.de.