Summer term 2022
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Linear and Integer Optimization
Assignment Sheet 3
Inofficial English Translation

1. For $A \in \mathbb{R}^{m \times n}$, $c \in \mathbb{R}^n$ and $b = (b_1, \dots, b_m) \in \mathbb{R}^m$ let $x^* \in \mathbb{R}^n$ be an optimum solution of the LP $\max\{c^t x \mid Ax \le b\}$. Moreover, let $\tilde{b} = (\tilde{b}_1, \dots, \tilde{b}_m) \in \mathbb{R}^m$, and let $\tilde{x} \in \mathbb{R}^n$ be a vector with $A\tilde{x} \le \tilde{b}$. Prove that $\tilde{x}$ is an optimum solution of the LP $\max\{c^t x \mid Ax \le \tilde{b}\}$ if $a_i^t \tilde{x} < \tilde{b}_i$ implies $a_i^t x^* < b_i$ for all $i \in \{1, \dots, m\}$ (where a_i^t is the i-th row of A). (5 points)

2. Consider the following primal-dual pair of linear programs: $\max\{c^t x \mid Ax \le b, x \ge 0\}$ and $\min\{b^t y \mid A^t y \ge c, y \ge 0\}$. Suppose that $\{x \in \mathbb{R}^n \mid Ax \le b, x \ge 0\}$ is a non-empty polytope. Show that there is a feasible dual solution y with $y > 0$ and $A^t y > c$. (5 points)

3. Let P be a polyhedron with $\dim(P) = d$ and F a face of P with $\dim(F) = k \in \{0, \dots, d-1\}$. Show that there are faces $F_{k+1}, F_{k+2}, \dots, F_{d-1}$ of P with
   i) $F \subseteq F_{k+1} \subseteq F_{k+2} \subseteq \dots \subseteq F_{d-1} \subseteq P$,
   ii) $\dim(F_{k+i}) = k + i$ for $i \in \{1, \dots, d-k-1\}$. (5 points)

4. Prove or disprove the following statement: If $X, Y \subseteq \mathbb{R}^n$ are polyhedra, then $X + Y$ is a polyhedron. (5 points)

Due date: Thursday, April 28, 2022, before the lecture in the lecture hall.