Prof. Dr. Stephan Held
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Exercise Set 9

Exercise 9.1. Provide an instance of the Simple Global Routing Problem which admits a fractional solution, but no feasible integral solution. Your instance has to satisfy $w(N, e) \leq u(e)$ for each net N and edge e.

(5 points)

Exercise 9.2. Let (G, H) be a pair of undirected graphs on $V(G) = V(H)$ with capacities $u : E(G) \to \mathbb{R}_+$ and demands $b : E(H) \to \mathbb{R}_+$. A *concurrent flow* of value $\alpha > 0$ is a family $(x^f)_{f \in E(H)}$ where x^f is an s-t-flow of value $\alpha \cdot b(f)$ in $(V(G), \{(v, w), (w, v) \mid \{v, w\} \in E(G)\})$ for each $f = \{t, s\} \in E(H)$, and

$$\sum_{f \in E(H)} x^f\big((v, w)\big) + x^f\big((w, v)\big) \leq u(e)$$

for all $e = \{v, w\} \in E(G)$. The Maximum Concurrent Flow Problem is to find a concurrent flow with maximum value $\alpha > 0$.

Prove that the Maximum Concurrent Flow Problem is a special case of the Min-Max Resource Sharing Problem. Specify how to implement block solvers.

(5 points)

Exercise 9.3. Prove that the number of oracle calls after $t \in \mathbb{N}$ phases of the core Resource Sharing Algorithm is bounded by

$$t|\mathcal{C}| + \frac{|\mathcal{R}|}{\epsilon} \ln \frac{||y^{(t)}||_1}{|\mathcal{R}|}.$$

Hint: Proceed similarly to the proof of Lemma 5.11 in the lecture notes.

(5 points)

Exercise 9.4. Let $G = (A \dot{\cup} B, E)$ be a bipartite graph. Assume that there is a matching covering A. Let $\varepsilon > 0$. Use the Resource Sharing Algorithm to find variables $(x_e)_{e \in E} \in [0, 1]^{E(G)}$ that satisfy

$$\sum_{e \in \delta(v)} x_e = 1 \qquad \forall v \in A, \qquad \sum_{e \in \delta(w)} x_e \leq 1 + \varepsilon \qquad \forall w \in B$$

within a running time of $\mathcal{O}(|E| \frac{\ln |B|}{\varepsilon^2})$.

(5 points)

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Deadline: June 21, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss22/chipss22_ex.html`

In case of any questions feel free to contact me at blankenburg@or.uni-bonn.de.