Exercise Set 9

Exercise 9.1. Consider the restriction $\mathcal{P}$ of the unweighted Vertex Cover Problem to graphs where the maximum degree of every vertex is bounded by a constant B.

Let $\varepsilon > 0$. Show: If there exists a polynomial time approximation algorithm for the Steiner Tree Problem with performance ratio $1 + \epsilon$, then there exists a polynomial time approximation algorithm for problem $\mathcal{P}$ with performance ratio $1 + (B + 1)\epsilon$.

(4 points)

Exercise 9.2. Let $G = (V, E)$ be a graph with non-negative edge costs, and let $S \subseteq V$ and $R \subseteq V$ be disjoint vertex sets ("senders" and "receivers"). Consider the problem of finding a minimum cost subgraph of G that contains a path connecting each receiver to a sender.

(a) Prove that the restriction of this problem to instances with $S \cup R = V$ is in P.

(b) Prove that this problem is NP-hard and give a 2-factor approximation algorithm.

(2+2 points)

Exercise 9.3. Give an $\mathcal{O}(n^3t^2)$ algorithm for the Steiner Tree Problem in planar graphs with all terminals lying on the outer face, where n is the number of vertices and t the number of terminals.

(Hint: Modify the Dreyfus-Wagner algorithm.)

(4 points)

Exercise 9.4. Consider the following algorithm for the STEINER TREE PROBLEM with 3 terminals v_1, v_2 and v_3: Find a shortest path P between v_1 and v_2 and let a be the distance of v_3 to P. Then find a vertex z minimizing $\sum_{i=1}^{3} dist(v_i, z)$ under the conditions

(i) $dist(v_i, z) \leq dist(v_1, v_2)$ for $i \in \{1, 2\}$ and

(ii) $dist(v_3, z) \leq a$.

The algorithm returns the union of the shortest paths from z to the terminals. Show that the algorithm needs $\mathcal{O}(|E| + |V| \log(|V|))$ time and works correctly.

(4 points)

Deadline: Tuesday, June 4$^{\text{th}}$, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss19/appr_ss19_ex.html`

In case of any questions feel free to contact me at rockel@or.uni-bonn.de.