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Exercise Set 8

We want to analyze Shallow-Light trees.

Definition. For a undirected graph G, a root $r \in V(G)$ and a metric length function $d : E(G) \to \mathbb{R}_{\geq 0}$, we denote for $s, t \in V(G)$ by $dist_{G,d}(s,t)$ the length of a shortest s-t-path w.r.t. d in G.

Definition. A (α, β)-Shallow-Light tree (SLT) for an undirected graph G with metric distances $d : E(G) \to \mathbb{R}_{\geq 0}$ and a root $r \in V(G)$, is a spanning tree T in G with cost at most $\alpha \cdot \mathrm{MST}(G)$ and for each $v \in V(G)$, the unique r-v-path in T has length at most $\beta \cdot dist_{G,d}(r,v)$.

Exercise 8.1.

(i) Let $\beta > 1$. Show that finding $(1, \beta)$-SLTs is NP-hard.
(Hint: Use a reduction from 3-SAT.)

(ii) For given $\beta > 1$ and $1 \leq \alpha < 1 + \frac{2}{\beta - 1}$, construct (G, d, r), such that there is no (α, β)-SLT for G.

(iii) Let $\beta > 1$ and $1 \leq \alpha < 1 + \frac{2}{\beta - 1}$. Show that finding (α, β)-SLTs is NP-hard.
(Hint: Use the graph from (ii) to modify an instance of the problem from (i) in order to reduce (i) to this problem.)

(3+2+2 points)

The above bounds are tight:

Exercise 8.2. Give a polynomial time algorithm that computes a $(1+\frac{2}{\beta-1}, \beta)$-SLT.

(Hint: Start with an minimum spanning tree and replace excessively long paths by shortest paths.)

(5 points)

Deadline: Tuesday, May 28$^{\text{th}}$, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss19/appr_ss19_ex.html`

In case of any questions feel free to contact me at rockel@or.uni-bonn.de.