Exercise Set 6

Exercise 6.1. Prove that for any fixed $k \in \mathbb{N}$ the directed component LP for the problem of finding a k-restricted Steiner tree can be solved in polynomial time.

(5 points)

Exercise 6.2. Let T denote the terminal set in the Steiner Tree Problem and let $r \in T$ be an arbitrarily chosen root. Let

$$\mathrm{LP} = \min\left\{c(x) : \sum_{e\in\delta(U)} x_e \geq 1 \text{ for } U \subseteq V \setminus \{r\} \text{ with } U \cap T \neq \emptyset, x \geq 0\right\}.$$

Now we replace every edge $\{v, w\}$ by two directed edges (v, w) and (w, v) (with cost $c(\{v, w\})$. Consider the following LP:

$$\mathrm{BCR} = \min\left\{c(x) : \sum_{e\in\delta^-(U)} x_e \geq 1 \text{ for } U \subseteq V \setminus \{r\} \text{ with } U \cap T \neq \emptyset, x \geq 0\right\}.$$

(a) Prove that the value BCR is independent of the choice of the root $r \in T$.

(b) What is the supremum of $\frac{\mathrm{BCR}}{\mathrm{LP}}$ over all instances (with $\mathrm{LP} \neq 0$)?

(5 points)

Exercise 6.3. Let $G = (V, E)$ be an undirected graph. For a partition $\mathcal{P}$ of the vertex set V let

$$\delta(\mathcal{P}) := \{e : e \in \delta(U) \text{ for some } U \in \mathcal{P}\}.$$

Prove

$$\left\{x \in [0,1]^E : \sum_{e\in E} x_e = |V(G)| - 1, \sum_{e\in E(G[X])} x_e \leq |X| - 1 \text{ for } \emptyset \neq X \subsetneq V(G)\right\}$$

$$= \left\{x \in [0,1]^E : \sum_{e\in E} x_e = |V(G)| - 1, \sum_{e\in\delta(\mathcal{P})} x_e \geq |\mathcal{P}| - 1 \text{ for every partition } \mathcal{P} \text{ of } V\right\}.$$

(5 points)

Exercise 6.4. Consider the following LP relaxation for the minimum spanning tree problem:

$$\min \left\{ c(x) : x \in [0,1]^E, \sum_{e \in \delta(X)} x_e \geq 1 \text{ for } \emptyset \neq X \subsetneq V(G) \right\}.$$

Show that the integrality gap of this LP relaxation is 2.

(Do not use Jain's algorithm for the Survivable Network Design Problem.)

(5 points)

Deadline: Thursday, June 7th, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss18/appr_ss18_ex.html`

In case of any questions feel free to contact me at traub@or.uni-bonn.de.