Exercise Set 1

Exercise 1.1. Consider the following procedure for (unweighted) Minimum Vertex Cover: Given a graph G, compute a DFS tree for every connected component. Return all vertices with non-zero out-degree in the tree. Show that this is a 2-approximation algorithm.

(4 points)

Exercise 1.2. The LP relaxation of the Minimum Weight Vertex Cover Problem is

$$\min\{cx : M^T x \geq 1, x \geq 0\}$$

where M is the incidence matrix of an undirected graph G and $c \in \mathbb{R}_+^{V(G)}$. A *half-integral* solution for this relaxation is one with entries 0, $\frac{1}{2}$ and 1 only.

(a) Show that the above LP relaxation of the Minimum Weight Vertex Cover Problem always has a half-integral optimum solution.

(b) Use this to obtain a 2-approximation algorithm. Is the analysis tight?

(4+2 points)

Exercise 1.3. This exercise develops a combinatorial algorithm for finding an optimal half-integral vertex cover.

Given an undirected graph $G = (V, E)$ and a non-negative cost function c on the vertices, obtain a bipartite graph $H(V' \,\dot{\cup}\, V'', E')$ as follows. Corresponding to each vertex $v \in V$ there are vertices $v' \in V'$ and $v'' \in V''$, each of cost $\frac{1}{2}c(v)$. Corresponding to each edge $(u, v) \in E$, there are two edges $(u', v''), (u'', v') \in E'$.

Show that a vertex cover in H can be mapped to a half-integral vertex cover in G preserving total cost and vice versa. Use the fact that an optimal vertex cover in a bipartite graph can be found in polynomial time to obtain an optimal half-integral vertex cover in G.

(4 points)

Exercise 1.4. Formulate linear-time 2-factor approximation algorithms for the following optimization problems and prove performance ratio as well as running time:

(a) Given an undirected, unweighted graph G, determine $v, w \in V(G)$ such that their distance is maximum.

(b) Given a directed graph G with non-negative edge weights, find an acyclic subgraph of maximum weight.

(c) Maximum-Satisfiability: Given an instance for Satisfiability, determine an assignment of truth values satisfying the maximum number of clauses.

(6 points)

Deadline: Tuesday, April 17$^{\text{th}}$, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ss18/appr_ss18_ex.html`

In case of any questions feel free to contact me at traub@or.uni-bonn.de.