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Exercise Set 2

Exercise 2.1. A Boolean function $f \in B_n$ *depends essentially* on all its variables if for every $1 \leq i \leq n$ the subfunctions $f|_{x_i=0}$ and $f|_{x_i=1}$ are different.

Let $f \in B_n$ be a function that essentially depends on all its variables. Show:

(a) $S_{B_2}(f) \geq n - 1$,

(b) $D_{B_2}(f) \geq \lceil \log_2 n \rceil$.

(5 points)

Exercise 2.2. For $f \in B_n$ let $S_{\text{DNF}}(f)$ be the minimum number of literals needed in any DNF representation of f. Show that $\max\{S_{\text{DNF}}(f) \mid f \in B_n\} \in \Omega(2^n)$.

(5 points)

Exercise 2.3. Let $f \in B_n$ be a Boolean function given as an oracle (i.e. for each $x \in \{0, 1\}^n$ the value $f(x)$ can be computed in $\mathcal{O}(1)$ time). Show that the set $PI(f)$ of all prime implicants can be computed in $\mathcal{O}(n^2 3^n)$ time.

(5 points)

Exercise 2.4. Show that the multiplication of two n-bit numbers can be implemented with a Boolean circuit over B_2 of size $\mathcal{O}(n^2)$, depth $\mathcal{O}(\log^2 n)$ and maximum fanout 2.

(5 points)

Deadline: May 4^{th}, before the lecture. The websites for lecture and exercises can be found at

`http://www.or.uni-bonn.de/lectures/ss17/chipss17.html`

In case of any questions feel free to contact me at ochsendorf@or.uni-bonn.de.